Time-Dependent Behaviour of Carbon Fibre Reinforced Laminates

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Abstract: On the basis of hereditary mechanics of solids relations methods of constitutive equations construction have been suggested. The main relations of algebra of resolvent operators are presented. Interrelation of number of material functions characterising rheological properties of carbon fibre reinforced composite laminates has been shown. With the help of identification methods the parameters of the constitutive equations were calculated which allow describing creep, relaxation and variable loading of specimens of carbon fibre reinforced composite laminates. The satisfactory agreement of predicted and experimental data was obtained. It is possible within the scope of the approach to take into account the influence of nonlinear properties on time-dependent behaviour of carbon fibre reinforced plastics.

Key Words: Resolvent operator, Constitutive equation, Creep, Relaxation.

1. Introduction

Structural members of carbon fibre reinforced composites are widely used in aerospace applications and are of great importance in describing the main regularities of time-dependent behavior of the laminates. It is obvious that time-dependent properties of fibrous laminated composites, to a large extent, are defined by the properties of polymer matrix and its adhesion to the fibre and appear under off-axis loading. To estimate time-dependent behaviour of carbon-fibre reinforced laminated composites, analysis of creep strain of angle ply composites to has been performed (Charentenay and Zaidi, 1982), (Deng et all, 2003), (Ma et all, 1997), (Potter, 1974). The most pronounced effect of creep is on the test of $[\pm 45]$ lay-up which is obviously connected with shear properties in the plane layer. In particular, an analysis of the experimental data of tensile loading of $[\pm 45]_{2s}$ lay-up carbon fibre reinforced samples has shown that rheological and nonlinear properties were determined by shear properties in the plane of the layer (Charentenay and Zaidi, 1982), (Potter, 1974) and creep strain was approximated by the Findley equation (Findley et al., 1976) by using power approximation. The power approximation was used in (Charentenay and Zaidi, 1982), (Deng et al., 2003), (Guedes et al., 1998), (Ma et al., 1997) creep of carbon-epoxy $[\pm 45]_{2s}$ was approximated by power function. Also, in order to satisfy some physical restrictions the time exponent parameter of the equation must lie between 0 and 1. Moreover, for carbon fibre reinforced composite laminates the parameter is close 0.1. Time-dependent properties are also dependent on strain rate (Guedes et al., 1998), (Hsiao and Daniel, 1998), (Potter, 1974). Constitutive equations of viscoelasticity allow analysis of the interrelation of time-dependent behaviour under different histories of loading (Dumansky and Strekalov, 1999), (Korontzis and Vellios, 2000), (Oza et all, 2003). The first stage of viscoelastic behaviour investigation is the estimation of elastic properties of the material. Elastic body strain instantly follows the stress and their time dependencies are similar. In the viscoelastic body, the effect of retardation takes place and is of great importance in dividing elastic and time-dependent strain. In measuring elastic properties we shall reduce the amount of time-dependent strain decreasing the time of loading and follow (Potter, 1974) to determine elastic characteristics that are necessary to load the specimen to failure within a few seconds. It should be noted that the elastic characteristics do not coincide with the characteristics defined under quasi-static loading.

When predicting the mechanical properties of carbon-fibre reinforced composite laminate, as a base are the properties of the layer. The majority of the papers of World-Wide Failure Exercise (ed: V.J. Hinton et al., 2004) devoted to mechanical behaviour of polymer composites under quasi-static loading are based on elastic properties of the layer and classical lamination theory relations. The inverse problem concluding in defining the elastic properties of the layer on the base of composite laminates experimental data is considered in (Zinoviev and Tairova, 1995) in which a method of identification of the layer's elastic properties on angle ply lay-up samples testing results. The application of classical lamination theory to describe time-dependent behaviour of carbon fibre reinforced laminates was considered in (Guedes et al., 1998), (Korontzis and Vellios., 2000), (Dumansky and Tairova, 2007, 2008). Finally, it is our opinion, that particularly promising, is the approach of using hereditary operator representation in constitutive equations. The theory of resolvent operators and its application to hereditary solid mechanics is described in (Rabotnov, 1979).

2. Elastic properties of the layer

The tension test of cross-ply carbon reinforced plastics based on viscoplastic resin had been the subject of inves-

tigation. Flat specimens of *[0]*$_4$, *[±20]* $_{2s}$, *[±40]* $_{2s}$, *[±50]* $_{2s}$, *[±70]* $_{2s}$, *[90]*$_4$ lay-ups were tested under some triangular cycles of quasi-static tensile strain. Stress-strain diagrams under quasi-static uniaxial tension with unloading to 0.3-0.7 of failure stress in longitudinal and transverse directions were obtained. These diagrams are shown in Fig. 1. Elastic moduli and Poisson's ratio were determined within linear range of stress-strain diagrams using identification method (Zinoviev and Tairova, 1995).
All the diagrams apart from *[0]*$_{2s}$, *[90]*$_{2s}$ lay-ups revealed nonlinear viscoelastic properties especially significant on *[±40]* $_{2s}$, *[±50]* $_{2s}$, lay-ups.

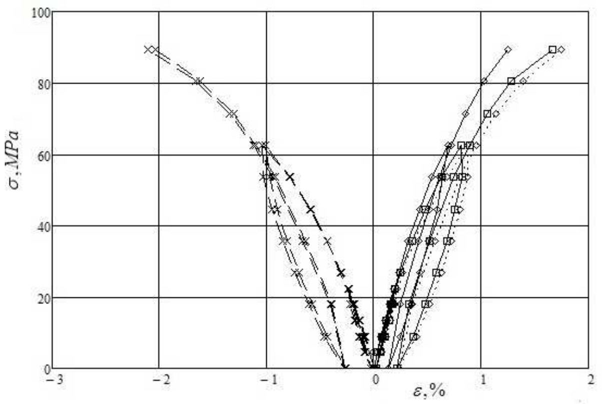

Fig.1. Stress-strain diagrams for *[±40]* $_{2s}$ lay up.

To determine the technical characteristics of the layer the minimization of the following residual function was performed as follows

$$\sum_k \left(\varepsilon_k^{\exp} - \varepsilon_k^{calc}\left(E_1, E_2, G_{12}, \nu_{12}\right)\right)^2 \to \min \tag{1}$$

The values (Dumansky and Tairova, 2007) appeared to be equal: $E_1 = 150$ MPa, $E_2 = 3.95$ MPa, $G_{12} = 2.39$ MPa, $\nu_{12} = 0.315$.

3. Algebra of resolvent operators

The operator form of the constitutive hereditary equation under uniaxial loading can be written in the form

$$E\varepsilon = \sigma + K^*\sigma = \left(1 + K^*\right)\sigma, \tag{2}$$

where $K^*\sigma = \int_0^t K(t-\tau)\sigma(\tau)\,d\tau$ is a kernel of the operator.
Constitutive equation (2) (Rabotnov, 1979) can be inversed and specified by

$$\sigma = E\left(\varepsilon - R^*\varepsilon\right) = E\left(1 - R^*\right)\varepsilon, \tag{3}$$

where operator R^* is resolvent in relation to the operator K^*.
Substituting the expression for σ from (3) into (2) yields the equation for the interrelation between the initial operator and its resolvent

$$\left(1 + K^*\right)^{-1} = 1 - R^*. \tag{4}$$

Solving equation (4) the common expression for the resolvent operator can be represented by

$$R^* = \frac{K^*}{1 + K^*} = K^* - K^{*2} + K^{*3} - \tag{5}$$

The series in relation (5) is Neumann series for the resolvent (Rabotnov, 1979) Rabotnov's fraction-exponential function was obtained as the resolvent of Abel's operator using the expression for kernel of the operators' multiplication

$$N(t-\tau) = \int_\tau^t M(t-x)L(x-\tau)\,dx. \tag{6}$$

For Abel's kernel $K(t) = I_\alpha(t) = \dfrac{t^\alpha}{\Gamma(1+\alpha)}, -1 < \alpha < 0$, the expression for Abel's operator is given by $\mathrm{I}_\alpha^* \cdot 1 = \mathrm{I}_\alpha^* = \dfrac{t^{1+\alpha}}{\Gamma(2+\alpha)}$, 1 is a unity function.
With the aid of formula (6) the power of Abel's operator can be written as (Rabotnov, 1979)

$$I_\alpha^{*m} = I_{m-1+m\alpha}^*. \tag{7}$$

Substituting in place of the operator K^* in relation of Neuman series fraction-exponential function takes the following form

$$Z_\alpha^*(-\beta) = -\beta \mathrm{I}_\alpha^* + \beta^2 \mathrm{I}_\alpha^{*2} - \beta^3 \mathrm{I}_\alpha^{*3} + \dots. \tag{8}$$

Using (Rabotnov, 1979) the explicit form of Rabotnov's fraction-exponential function can be written as

$$\mathrm{Z}_\alpha^*(-\beta) \cdot 1 = t^{1+\alpha} \sum_{n=0}^{\infty} \frac{\left(-\beta t^{1+\alpha}\right)^n}{\Gamma\left[1 + (1+\alpha)(1+n)\right]}, \tag{9}$$

where $\Gamma()$ is gamma-function. Series (9) converges at $\beta > 0$.
These are two main relations of the algebra of fraction-exponential functions

$$\begin{gathered} \left(1 - xZ_\alpha^*(y)\right)^{-1} = 1 + xZ_\alpha^*(y+x); \\ Z_\alpha^*(x) Z_\alpha^*(y) = \frac{1}{x-y}\left[Z_\alpha^*(x) - Z_\alpha^*(y)\right]. \end{gathered} \tag{10}$$

The particular cases of Rabotnov's fraction-exponential function are the following: $Z_\alpha^*(0) = I_\alpha^*$ is Abel's operator, $Z_0^*(-\beta)$ is exponential operator, and $Z_0^*(0)$ is integration operator. It should be noted that kernel as a sum of the exponential functions is also allow a resolvent operator.

4. Constitutive equations

It is well known that viscoelastic material can be identified the one of the number of material functions. In particular stress and strain in the viscoelastic material can be connected with the aid of Stieltjs convolutions, (Rabotnov, 1979)

$$\begin{gathered} \varepsilon(t) = J * d\sigma = \int_0^t J(t-\xi)\,d\sigma; \\ \sigma(t) = G * d\varepsilon = \int_0^t G(t-\xi)\,d\varepsilon \end{gathered}. \tag{11}$$

Then under creep and relaxation relations (11) can be written as

$$\begin{aligned}\varepsilon(t)&=J(t)\sigma;\\ \sigma(t)&=G(t)\varepsilon,\end{aligned}\qquad(12)$$

where $J(t)$, $G(t)$ are creep and relaxation functions.

Similar to (12) under strain and stress rate loading constitutive equations take the following form

$$\begin{aligned}\varepsilon(t)&=\eta(t)\dot{\sigma};\\ \sigma(t)&=\varsigma(t)\dot{\varepsilon}.\end{aligned}\qquad(13)$$

Using relations (12) and (13) we can describe strain and stress under piecewise loading. For loading which can be defined with the following forms

$$\begin{aligned}\sigma(t)&=\sum_{k=1}^{n}H(t-t_k)\Delta\sigma_k;\\ \varepsilon(t)&=\sum_{k=1}^{n}H(t-t_k)\Delta\varepsilon_k,\end{aligned}\qquad(14)$$

where $H(\ )$ is Heaviside unit function, $\Delta\sigma_k$, $\Delta\varepsilon_k$ are step-wise change of stress and strain. The corresponding change of strain and stress in consequence with (12) (Bugakov, 1973) can be represented as

$$\begin{aligned}\varepsilon(t)&=\sum_{k=1}^{n}J(t-t_k)\Delta\sigma_k;\\ \sigma(t)&=\sum_{k=1}^{n}G(t-t_k)\Delta\varepsilon_k.\end{aligned}\qquad(15)$$

Obviously similar relations can be obtained for relations (13)

$$\begin{aligned}\varepsilon(t)&=\sum_{k=1}^{n}\eta(t-t_k)\Delta\dot{\sigma}_k;\\ \sigma(t)&=\sum_{k=1}^{n}\varsigma(t-t_k)\Delta\dot{\varepsilon}_k\end{aligned}\qquad(16)$$

There are relations for connection between the material functions

$$\eta(t)=\int_0^t J(\xi)d\xi,\ \varsigma(t)=\int_0^t G(\xi)d\xi.\qquad(17)$$

These are in turn connected with creep and relaxation kernels by the following relations

$$\begin{aligned}J(t)&=\frac{1}{E}\left(1+\int_0^t K(\xi)d\xi\right);\\ G(t)&=E\left(1-\int_0^t R(\xi)d\xi\right).\end{aligned}\qquad(18)$$

5. Creep and relaxation *[±40]$_{2s}$* lay-up

Creep and relaxation of plane samples *[±40]$_{2s}$* lay-up were made on servo hydraulic testing machine Instron 8800. In the process of testing longitudinal and transverse strain were made on the base of 5 mm in 5 minutes interval. Clearly pronounced creep and relaxation of CFRP were observed. The level of load under creep was close to limit of elastic strain and equal to 65 MPa. The initial level strain of the relaxation was equal to 0.5%.

Experimental data were analyzed and treated to describe creep curve of *[±40]$_{2s}$* lay-up. Similar to (Charentenay and Zaidi, 1981) power law representing Abel's kernel of the constitutive equation was taken. The linear constitutive hereditary equation with Abels's kernel is given by

$$\varepsilon(t)=\frac{1}{E_0}\left(1+kI_\alpha^*\right)\sigma,\qquad(19)$$

where E_0 is instant modulus, k, α are parameters of the equation. In case of creep constitutive equation (19) can be rewritten as

$$\varepsilon(t)=\varepsilon_0\left(1+kI_\alpha^*\right)=\varepsilon_0\left(1+\frac{k}{\Gamma(2+\alpha)}t^{1+\alpha}\right),\qquad(20)$$

where ε_0 is instant elastic strain. The parameters of the constitutive equation were calculated by minimizing of the following expression

$$\sum_k\left(\varepsilon_k^{\exp}-\varepsilon_k^{calc}(E_0,k,\alpha)\right)^2\to\min.\qquad(21)$$

The parameters are as follow: $E_0=19900$ MPa, $\alpha=-0.894$, $k=0.873$ min$^{-(1+\alpha)}$. The prediction of creep strain shown in Fig. 2 is satisfactory.

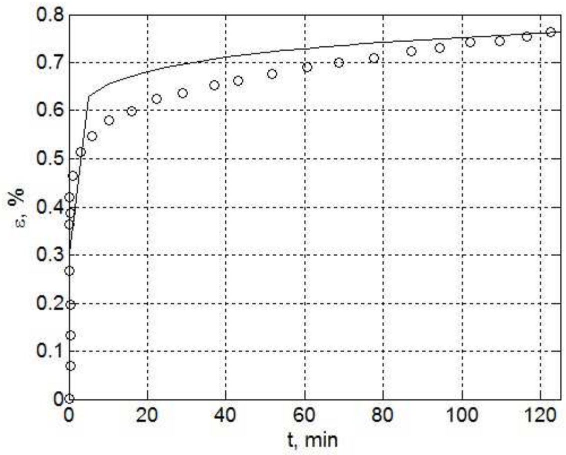

Fig. 2.

Using the above calculated parameter values the relaxation curve was obtained. The experimental values and predicted curve are shown in Fig. 3.

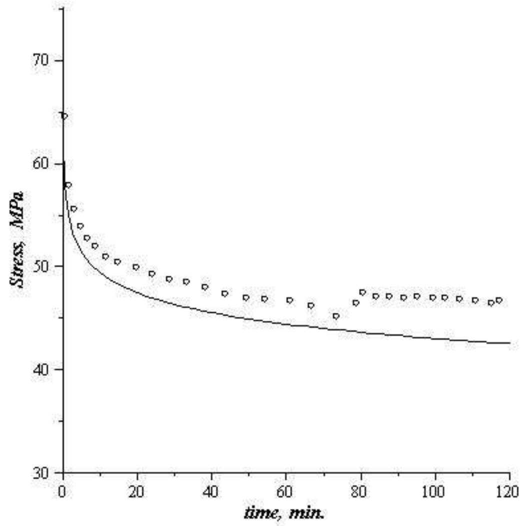

Fig. 3.

The linear constitutive equation is obtained by use of relation (10) and can be written as

$$\sigma = E_0\left(1 - kZ_\alpha^*(-k)\right)\varepsilon . \tag{22}$$

Using the explicit form of fraction-exponential function (9) representation constitutive equation for relaxation is as follows

$$\sigma = E_0\varepsilon_0\left(1 - kt^{1+\alpha}\sum_{n=0}^{\infty}\frac{\left(-\beta t^{1+\alpha}\right)^n}{\Gamma\left[1+(1+\alpha)(1+n)\right]}\right). \tag{23}$$

6. Time-dependent behaviour under variable loading

Experimental studies under variable loading were performed for *[±40]* $_{2s}$ specimens. There were four trapezoidal cycles of strain and the corresponding strain in longitudinal and transverse directions were measured. The results of the testing is shown in Fig. 4.

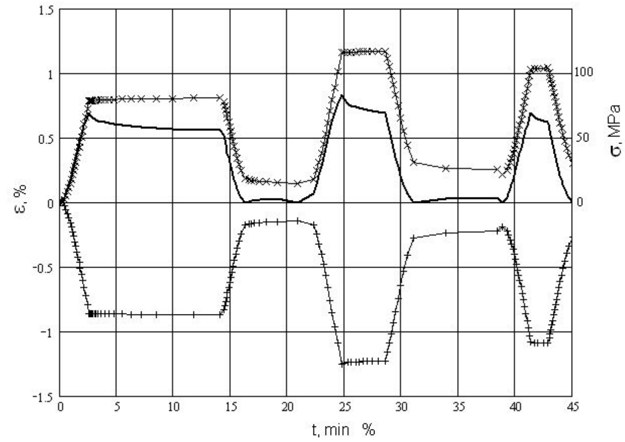

Fig. 4. Stress response of *[±40]* $_{2s}$ lay up to strain input.

Within the permanent strain ranges there is relaxation of stress. Isochronic curves of *[±40]* $_{2s}$ lay up are shown in Fig 2. In Fig. 2 one can see that hysteresis effects are determined principally by rheological properties. These are reflected by vertical parts of the curves in Fig. 5. The residual strain determining the shift of the diagrams, to some extent, depends on the horizontal shift dependent on the time of recovery.

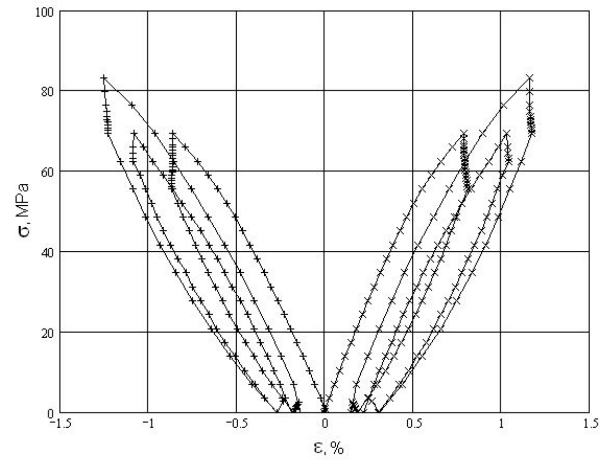

Fig. 5. Isochronic curves of carbon fibre reinforced composite laminates *[±40]* $_{2s}$ lay up.

To describe stress response under trapezoidal strain input it is convenient to represent strain in strain rate∼time coordinates.

From all the array of the strain values only eleven points were chosen in which the strain values have derivative discontinuity. In this case the strain rate is given by

$$\dot{\varepsilon} = \sum_{k=1}^{n} H\left(t - t_k\right)\Delta\dot{\varepsilon}_k . \tag{24}$$

And using the second relation from (16) the stress dependence can be described. In the case of Abel's creep kernel, the material function in the constitutive equation in (16) is taken as follows

$$\varsigma(t) = E\left(t - Z_\alpha^*(-\beta)\cdot t\right). \tag{25}$$

By integration of power functions in series (9) it is not hard to obtain the explicit form of material function $\varsigma(t)$. The explicit form of the operator yields

$$Z_\alpha^*(-\beta)\cdot t = t^{2+\alpha}\sum_{n=0}^{\infty}\frac{\left(-\beta t^{1+\alpha}\right)^n}{\Gamma\left[2+(1+\alpha)(1+n)\right]}. \tag{26}$$

Using constitutive equation (26) and the second equation in (16) it is possible to describe the stress-strain diagram under loading shown in Fig. 4

. The comparison of experimental data and calculated values is presented in Fig. 6. Strain rate increment is as follows: $\Delta\dot{\varepsilon}_k = \frac{\Delta\varepsilon_{k+1}}{\Delta t_{k+1}} - \frac{\Delta\varepsilon_k}{\Delta t_k}$.

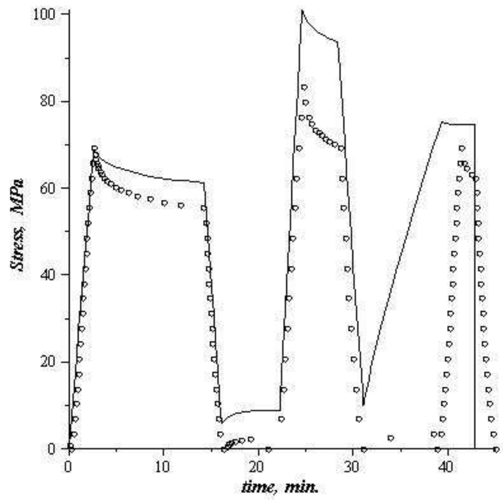

Fig. 6.

The difference in Fig. 5 can be explained by the presence of nonlinear effects especially in the second and third cycles of loading in which the level of stresses increase the linear limit.

7. Nonlinear behaviour

Nonlinear effect can be taken into account by change of left part of constitutive equation (2) (Rabotnov, 1979) and rewritten as

$$\varphi(\varepsilon) = \left(1 + K^*\right)\sigma , \tag{27}$$

where $\varphi(\varepsilon)$ is instant strain curve which is defined by data treatment. The possible way of its representation is as follows

$$\varphi(\varepsilon) = E_0\varepsilon - \Delta E_0 \cdot (\varepsilon - \varepsilon_1) \cdot H(\varepsilon - \varepsilon_1), \qquad (28)$$

where ΔE_0 is change of instant modulus E_0 at strain ε_1. Substituting (28) into (27) and by use of (3) the constitutive equation for stress specified by

$$\sigma = E_0 (1 - R^*) \varepsilon - \Delta E_0 R^* (\varepsilon - \varepsilon_1). \qquad (29)$$

An application of such approach to carbon fibre reinforced composite laminates under quasi-static loading was considered in (Dumansky et al., 2011).

8. Results and discussion

The possibility of application of mechanics of hereditary solids in describing of time-dependent behaviour of carbon fibre reinforced composite laminates was shown. The use of algebra of resolvent operators and theory of generalized functions allow significant simplifying the construction the constitutive equations. Fraction-exponential function combining properties of power and exponential functions can be successfully used to characterize rheological properties of carbon fibre reinforced composites. A systematic investigation was conducted of time-dependent behaviour of carbon fibre reinforced laminates. A preliminary estimation of the singularity parameter α which is connected with the time exponent in the Findley's law is equal to -0.9. The possibility of model generalization to take into account the nonlinear behaviour of carbon fibre reinforced composite laminates has been shown.

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