The prediction of viscoelastic properties of layered composites on example of cross ply carbon reinforced plastic

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Abstract—**Structure– phenomenological hereditary model for prediction of viscoelastic properties of layered polymer composites based on hereditary mechanics and theory of laminated plates relationships was elaborated. Interrelated creep and relaxation constitutive equations of layered composite was derived on the basis of algebra of resolvent operators and matrix algorithms. On example of cross-ply carbon reinforced plastic the prediction of viscoelastic properties was shown.**

Keywords: carbon reinforced plastic, hereditary operator, constitutive equations, resolvent operator, matrix resolvent

1 Introduction

Unidirectional fiber polymer composites show evidence of significant viscoelastic properties under shear and the last (ones) in turn induce viscoelastic properties of layered laminates. The most common approach to description of long term loading is in use of constitutive hereditary equations [1, 2, 3, 8, 10, 12] or relationships following from them [6, 4, 11]. Engineering practice is confined by creep, creep and recovery or constant rate tests of cross-ply composites to define viscoelastic properties. Saw-shaped cycle of loading by strain considered in [10]. It is of value to choose of suitable approximating function that is a kernel of hereditary equation. For the most part these are power-law [11, 13, 6, 4] or exponential creep [10, 11, 12] relations. To improve data approximation in [11] linear combination of power-law and exponential functions was used. Rabotnov fraction exponential function [1] possesses properties of power-law (shorter time) and exponential (longer time) functions. Nonlinear viscoelastic behavior is generally represented as the extension of linear models [1, 3, 8, 12], taking into account some irreversible effects. Of great importance is interrelation of creep and relaxation properties [1, 3, 7, 8]. From analytical point of view, interrelation of viscoelastic properties implies inverse of the constitutive equations or establishing interrelation between creep and relaxation functions. The procedure of the constitutive equations inverse can be established by means of Laplace transform or resolvent operator. This study in terms of resolvent operator properties was intended to obtain an efficient approach of establishing interrelation between the constitutive equations describing the viscoelastic properties of layered laminates. Comparisons of theoretical results were made with experiments conducted on cross-ply carbon reinforced plastics.

2 Experimental Procedure

Tension test of cross-ply carbon reinforced plastics based on viscoplastic resin had been the subject of investigation. Flat specimens of $[0]_4$, $[\pm 10]_4$, $[\pm 20]_4$, $[\pm 40]_4$, $[\pm 50]_4$, $[\pm 70]_4$ $[90]_4$ lay-ups were tested under some saw-shaped cycles of quasi-static tensile strain. Stress-strain diagrams with unloading to 0.3-0.7 of failure stress in longitudinal and transverse directions were obtained. Elastic moduli and Poisson's ratio were determined within linear range of stress-strain diagrams using identification method [5]. All the diagrams apart from $[0]_4$ and $[90]_4$ lay-ups revealed nonlinear viscoelastic properties especially significant on $[\pm 40]_4$, $[\pm 50]_4$ lay-ups. Stress-strain diagrams under quasi-static tensile strain in longitudinal and transverse directions are shown in Figure 1. On the

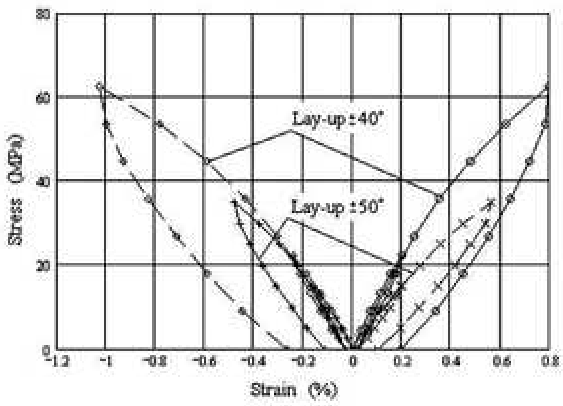

Figure 1: Stress-strain diagrams of cross-ply cfrp.

base of data analysis it was supposed that rheology of unidirectional carbon fibre reinforced plastic depends on only of shear strain. To check the hypothesis specimens of $[\pm 40]_4$ were tested together with registration of strain- and stress-time values. The results of the experiment are shown in Figure 2.

3 Model Description

The structure-phenomenological model is based on assumption that unidirectional layer has viscoelastic properties only under shear and are described by hereditary constitutive equation:

$$\gamma_{12} = \frac{1}{G_{12}^0}\left(1 + K^*\right)\tau_{12}, \tag{1}$$

where $K^*\tau_{12} = \int\limits_0^t K\left(t-\xi\right)\tau_{12}\left(\xi\right)d\xi$- hereditary linear operator, $K(t)$ - kernel of the operator, G_{12}^0 - shear instantaneous modulus [1] that is different from its quasi-static value and herein calculates by the data handling. As it follows from relationship (1) the shear modulus is a modulus where time of loading tends to zero. An acceptable assessment of instantaneous shear modulus may be obtained as insignificant amount of creep during a shear test. As shown in [9] it is necessary to load specimen to failure within a few seconds. As stated above in the orthotropy directions mechanical properties of the unidirectional layer follow to Hook's law. Then, the constitutive equations of the unidirectional layer may be written as

$$\left\{\begin{array}{c}\varepsilon_1\\ \varepsilon_1\\ \gamma_{12}\end{array}\right\} = \left[\begin{array}{ccc}\frac{1}{E_1} & -\frac{\nu_{21}}{E_2} & 0\\ -\frac{\nu_{12}}{E_1} & \frac{1}{E_2} & 0\\ 0 & 0 & \frac{1}{G_{12}^0}\left(1+K^*\right)\end{array}\right]\left\{\begin{array}{c}\sigma_1\\ \sigma_2\\ \tau_{12}\end{array}\right\}. \tag{2}$$

Matrix (2) can be inverted by inverse of the two diagonal blocks. The first one is the digital matrix, the second can be inverted with the aid of resolvent of hereditary operator [1]

$$\tau_{12} = \left(\frac{1}{G_{12}^0}\left(1+K^*\right)\right)^{-1}\gamma_{12} = G_{12}^0\left(1-R^*\right)\gamma_{12}, \tag{3}$$

where R^* - hereditary resolvent operator induced by operator K^*. The form of the inverted matrix is similar to (2) and in matrix form can be written as [1]

$$\left\{\sigma_{12}\right\} = \left[\mathbf{G_{12}^0} - \mathbf{G_{12}^*}\right]\left\{\varepsilon_{12}\right\}, \tag{4}$$

where $\{\sigma_{12}\}$, $\{\varepsilon_{12}\}$ - columns of strains and stresses, matrix $\mathbf{G_{12}^*}$ is as

$$\mathbf{G_{12}^*} = \left[\begin{array}{ccc}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & G_{12}^0\end{array}\right]R^* = \mathbf{G_{12}^0}R^*. \tag{5}$$

[1]Here and further matrices are bold typed

Using relationships of theory of laminated plates we can get stiffness matrix of multilayered plate:

$$\tilde{\mathbf{G}}_{\mathbf{xy}} = \mathbf{G_{xy}} - \mathbf{G_{xy}^*}, \tag{6}$$

where $\mathbf{G_{xy}} = \sum\limits_{k=1}^{n}\mathbf{T_k G_{12} T_k^T}\bar{h}_k$ - matrix of instantaneous stiffnesses, $\bar{h}_k = \frac{h_k}{\sum\limits_{k=1}^{n} h_k}$ - relative thickness of k-layer, T_k - matrix of transition of stresses under rotation of coordinate system. The same transformations are applied to obtain matrix $\mathbf{G_{12}^*}$. Hereditary operator R^* is a multiplier and thus the stiffness matrix becomes

$$\tilde{\mathbf{G}}_{\mathbf{xy}} = \mathbf{G_{xy}} - \mathbf{G_{xy}^0}R^*. \tag{7}$$

Compliance matrix of the laminated plate $\tilde{\mathbf{S}}_{\mathbf{xy}}$ is obtained by inverse of stiffness matrix $\tilde{\mathbf{G}}_{\mathbf{xy}}$. Following the procedure presented in the Appendix the compliance matrix is

$$\tilde{\mathbf{S}}_{\mathbf{xy}} = \left[\mathbf{I} + \mathbf{Q}\,\mathbf{diag}(\lambda_\mathbf{i}\mathbf{R}^*(\mu - \lambda_\mathbf{i}))\mathbf{Q^{-1}}\right]\mathbf{S_{xy}}. \tag{8}$$

Input characteristics of the proposed model are:

- Elastic properties of unidirectional layer, i.e. elastic moduli and Poisson's ratio;
- On basis of chosen kernel of the shear constitutive hereditary equation we define shear instantaneous modulus and parameters of the kernel;
- Conducting the necessary matrix calculations allow us the constitutive equations to be determined.

It should be noted that in the absence of temporal component the constitutive equations transform to the classical laminate theory relationships. By means of additional resolvent operator transformations the explicit form of the relaxation moduli and Poisson's ratio can be obtained.

4 Model of Application

The procedure of model application is demonstrated on cross-ply carbon fiber reinforced plastic with $\pm\theta$ lay-up. There were the following consequent steps:

1. On results of tension tests of $[0]_4$, $[\pm 10]_4$, $[\pm 20]_4$, $[\pm 40]_4$, $[\pm 50]_4$, $[\pm 70]_4$ $[90]_4$ lay-ups of carbon fiber reinforced plastics with the help of identification method elastic characteristics of the unidirectional layer were determined [5]: $E_1 = 150$ GPa, $E_2 = 3.95$ GPa, $G_{12} = 2.39$ GPa, $\nu_{12} = 0.315$. Shear modulus G_{12}^0 is defined together with the parameters of kernel of the constitutive equation on the base of time-dependent test results.
2. Viscoelastic properties of unidirectional carbon reinforced plastic were calculated on test results of cross-ply specimen $[\pm 40]_4$ lay-up under the strain varying

in time. The stress-time and strain-time relationships are presented in Figure 2. In relationship (1) Abel operator was taken as $K^* = \lambda I_\alpha^*$, where its kernel is $I_\alpha(t) = \frac{t^\alpha}{\Gamma(1+\alpha)}$. It is known [1] that resolvent of Abel operator is kernel of Rabotnov fraction exponential function $\epsilon_\alpha(-\lambda, t) = t^\alpha \sum_{n=0}^{\infty} \frac{(-\lambda t^{1+\alpha})^n}{\Gamma[(1+\alpha)(n+1)]}$, $\Gamma()$ - gamma-function. Relationship (3) in this case takes the following form

$$(1 + \lambda I_\alpha^*)^{-1} = 1 - \lambda \epsilon_\alpha^* (-\lambda), \qquad (9)$$

where effect fraction exponential function on f is

$$\epsilon_\alpha^* (-\lambda) \cdot f = \int_0^t \epsilon_\alpha (-\lambda, t - \xi) f(\xi)\, d\xi.$$

The resolvent of Abel operator is

$$R^* = \lambda \epsilon_\alpha^* (-\lambda). \qquad (10)$$

It is necessary to stress that the fraction exponential operator combines weak singularity of power-law Abel operator and asymptotical property of exponential operator [1]. At short times fraction exponential operator is similar to Abel operator

$$\epsilon_\alpha^*(-\lambda) \approx I_\alpha^*. \qquad (11)$$

And the following asymptotical property at $t \to \infty$ is

$$\epsilon_\alpha^*(-\lambda) \to \frac{1}{\lambda}. \qquad (12)$$

3. Most of the modes of long term loading in engineering can be approximated by step functions at $\sigma - t, \dot{\sigma} - t$ or $\varepsilon - t, \dot{\varepsilon} - t$ plots. The parameters of constitutive equation (1) were defined on test result of cross ply carbon reinforced plastic ± 40 lay up under two first step strain rate change. The two steps of strain presented in Figure 2 were written as

$$\dot{\varepsilon}_x(t) = \left\{ \begin{array}{ll} H(t)\dot{\varepsilon}_1, & 0 < t < t_1 \\ (H(t) - H(t - t_1))\dot{\varepsilon}_1, & t > t_1 \end{array} \right., \qquad (13)$$

where $H(t)$ - Heaviside function, t_1 - the point of strain rate change. Under uniaxial tension $\{\sigma_{xy}\} = \{\sigma_x, 0, 0\}^T$ with the aid of (7) the matrix form of the constitutive equations can be written as

$$\{\sigma_{xy}\} = \left(\mathbf{G_{xy}} - \mathbf{G^0_{xy}} R^*\right) \cdot t \cdot \{\dot{\varepsilon}_{xy}\}, \qquad (14)$$

where $\{\dot{\varepsilon}_{xy}\} = \left\{ \begin{array}{ccc} \dot{\varepsilon}_x & \dot{\varepsilon}_y & 0 \end{array} \right\}^T$. In system (14) there are two nonzero constitutive equations which can be rewritten as

$$\sigma_x = \left(g_{xx} - g^0_{xx} R^*\right) \cdot t \cdot \dot{\varepsilon}_x + \left(g_{xy} - g^0_{xy} R^*\right) \cdot t \cdot \dot{\varepsilon}_y;$$

$$0 = \left(g_{xy} - g^0_{xy} R^*\right) \cdot t \cdot \dot{\varepsilon}_x + \left(g_{yy} - g^0_{yy} R^*\right) \cdot t \cdot \dot{\varepsilon}_y, \qquad (15)$$

where $g_{xx}, g^0_{xx}, g_{xy}, g^0_{xy}, g_{yy}, g^0_{yy}$ - elements of matrices $\mathbf{G_{xy}}$ and $\mathbf{G^0_{xy}}$.
Excluding $\dot{\varepsilon}_y$ from (15), we have the following constitutive equation

$$\sigma_x = \left[g_{xx} - \frac{g^2_{xy}}{g_{yy}} - \left(g^0_{xx} - \frac{\left(g^0_{xy}\right)^2}{g^2_{yy}} \right) R^* \right] \cdot t \cdot \dot{\varepsilon}_x. \qquad (16)$$

Inserting (10) into (16) gives

$$\sigma_x = E_x \left(1 - \lambda_x \epsilon_\alpha^* (-\lambda)\right) \cdot t \cdot \dot{\varepsilon}_x, \qquad (17)$$

where $E_x = g_{xx} - \frac{g^2_{xy}}{g_{yy}}$ - instantaneous modulus of cross-ply composite in x direction, $\lambda_x = \frac{\lambda}{E_x}\left(g^0_{xx} - \frac{\left(g^0_{xy}\right)^2}{g^0_{yy}} \right)$. Denoting $E_x \left(1 - \lambda_x \epsilon_\alpha^* (-\lambda)\right) \cdot t = \eta_x(t)$, the constitutive equation corresponding to strain history (13) can be written

$$\sigma_x(t) = \left\{ \begin{array}{ll} \eta_x(t)\dot{\varepsilon}_x, & 0 < t < t_1 \\ (\eta_x(t) - \eta_x(t - t_1))\dot{\varepsilon}_x, & t > t_1 \end{array} \right.. \qquad (18)$$

Assuming that at short time of loading the fraction exponential function is close to Abel operator [1]. In this case the material function $\eta_x(t)$ takes a form

$$\eta_x(t) = E_x \left(1 - \frac{\lambda_x}{\Gamma(3 + \alpha)} t^{1+\alpha} \right) \cdot t. \qquad (19)$$

Test results show that as applied to carbon reinforced plastics singularity parameter α give stable results being equal to -0.9 [14]. Then E_x and λ_x are the parameters can be calculated by least-squares method

$$\sum_k \left(\sigma_k^{\exp} - \sigma_k(\alpha, E_x, \lambda_x)\right)^2 \to \min. \qquad (20)$$

As a result, the estimated values are: $E_x = 18.2$ GPa and $\lambda_x = 0.3521\ min^{-(1+\alpha)}$. With the values above-mentioned it was possible to determine the parameters characterizing shear viscoelastic properties of the unidirectional carbon reinforced plastic. They were the following: $\alpha = -0.9$ is the same as above-cited, $G^0_{12} = 3.6\ GPa$, $\lambda = 1.0465\ min^{-(1+\alpha)}$. Constitutive equation (1) for shear strain of unidirectional carbon reinforced plastic can be written

$$\gamma_{12} = \frac{1}{G^0_{12}} \left(1 + \lambda I_\alpha^*\right) \cdot \tau_{12}. \qquad (21)$$

Constitutive equation inverted to (1) can be obtained by substituting (10) into (3)

$$\tau_{12} = G^0_{12} \left(1 - \lambda \epsilon_\alpha^* (-\lambda)\right) \cdot \gamma_{12}. \qquad (22)$$

4. In general case the stress-strain diagrams can be given in parametric representation. Joining relationships (13) and (18) the two steps of stress-strain diagram plotted in Figure 2 can be described. In general case when the loading specified by strain rate

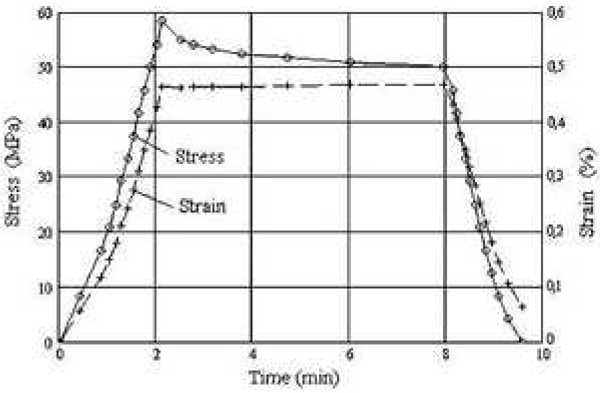

Figure 2: Stress and strain scaled diagrams versus time

the stress-strain diagrams can be represented as

$$\begin{cases} \dot{\varepsilon}\,(t) = \sum\limits_{i=1}^{n} H\,(t - t_i)\Delta\dot{\varepsilon}_i; \\ \sigma\,(t) = \sum\limits_{i=1}^{n} \eta\,(t - t_i)\Delta\dot{\varepsilon}_i, \end{cases} \tag{23}$$

where $\Delta\dot{\varepsilon}_i$ is rate strain change at i-step.
Similar relationships may be obtained where the other modes of temporal loading. The prediction of stress-strain diagram for the three steps of loading and comparison with test data are shown in Figure 3. In area of nonlinear strain a generalization of this model can be carried out by taking the additional hypotheses.

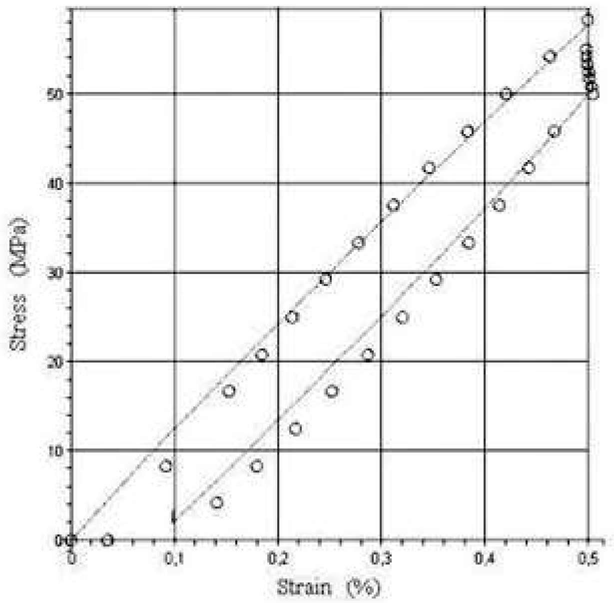

Figure 3: Prediction of strain-stress curve

5. Let us take a look at consecutive steps of matrices and resolvents calculation directly concerned with obtaining of the constitutive equations of cross ply carbon fiber reinforced plastic. Stiffness matrix of unidirectional layer is inverse to compliance matrix and equal to

$$\mathbf{G^0_{12}} = \mathbf{S^{0-1}_{12}} = \begin{pmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ & \frac{1}{E_1} & 0 \\ sym & & \frac{1}{G^0_{12}} \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 151.4 & 1.25 & 0 \\ & 3.96 & 0 \\ sym & & 3.6 \end{pmatrix} \tag{24}$$

Stiffness matrix of $[\pm 40]_4$ lay-up is equal to

$$\mathbf{G_{xy}} = 0.5\left(\mathbf{T}_\theta \mathbf{G^0_{12}} \mathbf{T}_\theta^{\mathbf{T}} + \mathbf{T}_{-\theta} \mathbf{G^0_{12}} \mathbf{T}_{-\theta}^{\mathbf{T}}\right)$$

$$= \begin{pmatrix} 56.9 & 34.8 & 0 \\ & 31.3 & 0 \\ sym & & 37.2 \end{pmatrix}, \tag{25}$$

where $\mathbf{T}_\theta = \begin{pmatrix} \mathrm{c}^2 & \mathrm{s}^2 & -\mathrm{sc} \\ \mathrm{s}^2 & \mathrm{c}^2 & \mathrm{sc} \\ -2\mathrm{sc} & 2\mathrm{sc} & \mathrm{c}^2 - \mathrm{s}^2 \end{pmatrix}$,
$c = \cos(\theta)$, $s = \sin(\theta)$.
Matrix $\mathbf{G^0_{xy}}$ calculation gives

$$\mathbf{G^0_{xy}} = \begin{pmatrix} 3.5 & -3.5 & 0 \\ & 3.5 & 0 \\ sym & & 0.109 \end{pmatrix} \tag{26}$$

Substituting (25), (26) into (7) and using that $K^* = \lambda I^*_\alpha$ we get matrix of relaxation moduli $\tilde{\mathbf{G}}_{\mathbf{xy}}$ of the layered plate.
Compliance matrix of the cross-ply specimen is equal to

$$\mathbf{S_{xy}} = \mathbf{G^{-1}_{xy}} = \begin{pmatrix} 0.051 & -0.056 & 0 \\ & 0.093 & 0 \\ sym & & 0.027 \end{pmatrix}. \tag{27}$$

Auxiliary matrix Λ is

$$\mathbf{\Lambda} = \mathbf{S_{xy}} \mathbf{G^0_{xy}} = \begin{pmatrix} 0.407 & -0.407 & 0 \\ -0.564 & 0.564 & 0 \\ 0 & 0 & 0.003 \end{pmatrix}.$$

Diagonal matrix of eigenvalues and matrix of eigenvectors are

$$\mathbf{D} = diag\,(\lambda_i) = diag\begin{pmatrix} 0 & 0.968 & 0.316 \cdot 10^{-2} \end{pmatrix};$$

$$\mathbf{Q} = \begin{pmatrix} -0.707 & 0.592 & 0 \\ -0.707 & -0.821 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

It should be made some common assessments of stiffness and compliance matrices. As stated above at initial point stiffness and compliance matrices coincide with its instantaneous values

$$\tilde{\mathbf{G}}_{\mathbf{xy}}\Big|_{t=0} = \mathbf{G}_{xy}; \qquad \tilde{\mathbf{S}}_{\mathbf{xy}}\Big|_{t=0} = \mathbf{S}_{xy}. \tag{28}$$

At infinity correct the following assessments

$$\mathbf{\tilde{G}_{xy}}\Big|_{\mathbf{t}=\infty} = \mathbf{G_{xy}} - \mathbf{G^0_{xy}} = \mathbf{G_{xy}}(\mathbf{I} - \mathbf{\Lambda});$$
$$\mathbf{\tilde{S}_{xy}}\Big|_{\mathbf{t}=\infty} = \left(\mathbf{I} + \mathbf{Q}\mathbf{diag}\left(\frac{\lambda_i}{\lambda+\lambda_i}\right)\mathbf{Q^{-1}}\right)\mathbf{S_{xy}}. \qquad (29)$$

Deviation mutually inverse matrices $\mathbf{I} - \mathbf{\Lambda}$ and $\mathbf{I} + \mathbf{Q}\mathbf{diag}\left(\frac{\lambda_i}{\lambda+\lambda_i}\right)\mathbf{Q^{-1}}$ of unit matrix allows perturbation caused by the time to be assessed. The calculated values are

$$\mathbf{I} - \mathbf{\Lambda} = \begin{pmatrix} 0.593 & 0.407 & 0 \\ 0.564 & 0.436 & 0 \\ 0 & 0 & 0.997 \end{pmatrix};$$

$$\mathbf{I} + \mathbf{Q}\mathbf{diag}\left(\frac{\lambda_i}{\lambda+\lambda_i}\right)\mathbf{Q^{-1}} \qquad (30)$$
$$= \begin{pmatrix} 1.202 & -0.202 & 0 \\ -0.280 & 1.280 & 0 \\ 0 & 0 & 1.003 \end{pmatrix}.$$

5 CONCLUSION

Structure-phenomenological model based on hereditary mechanics relationships allows the viscoelastic properties of the layered composites to be predicted on viscoelastic properties of the unidirectional layer. The anisotropy of viscoelastic properties of layered plates, and particularly, cross-ply carbon reinforced plastic can be described the derived constitutive equations. Operator expressions of creep and relaxation moduli by means of correspondence principle can be used to solve boundary value problems of viscoelasticity. The analysis of nonlinear viscoelastic properties can be made by modification of the proposed model. An example of model application to prediction of viscoelastic properties of cross-ply carbon reinforced plastic was demonstrated. Satisfactory agreement of calculated values and test results was shown.

Appendix

To accomplish inverse of the relaxation moduli matrix the following preliminary operations give

$$\mathbf{\tilde{G}^{-1}_{xy}} = \left(\mathbf{G_{xy}} - \mathbf{G^0_{xy}}\mathbf{R}^*\right)^{-1} =$$
$$\left[\mathbf{G_{xy}}\left(\mathbf{I} - \mathbf{G^{-1}_{xy}}\mathbf{G^0_{xy}}\mathbf{R}^*\right)\right]^{-1} = (\mathbf{I} - \mathbf{G^{-1}_{xy}}\mathbf{G^0_{xy}}\mathbf{R}^*)^{-1}\mathbf{G^{-1}_{xy}} \qquad (31)$$

where $\mathbf{I}$ - unit matrix. Denoting $\mathbf{\Lambda} = \mathbf{G^{-1}_{xy}}\mathbf{G^0_{xy}}$, and by the resolvent definition [1] we can perform the inverse procedure with the aid of Newman series matrix for expression in (31)

$$(\mathbf{I} - \mathbf{\Lambda}R^*)^{-1} = \mathbf{I} + \mathbf{\Lambda}R^* + \mathbf{\Lambda}^2 R^{*2} + ... \qquad (32)$$

The series in (32) is convergent if norm $\|\Lambda R^*\| < 1$. To inverse (32) we represent matrix $\mathbf{\Lambda}$ as: $\mathbf{\Lambda} = \mathbf{QDQ^{-1}}$, where $\mathbf{D}$ - diagonal matrix with eigenvalues λ_i on its diagonal and matrix $\mathbf{Q}$ with eigenvectors in columns.
Substituting decomposition of matrix $\mathbf{\Lambda}$ into Newman series matrix (32), we obtain

$$\mathbf{I} + \mathbf{\Lambda}R^* + ... = \mathbf{Q}\, diag(1 + \lambda_i R^* + ...)\mathbf{Q^{-1}}. \qquad (33)$$

If hereditary operator R^* has a parameter μ, i.e. $R^* = R^*(\mu)$, it induces the same resolvent operator with shift in the argument [1]. Then the relationship for diagonal matrix (33) can be reduced to

$$diag(1 + \lambda_i R^*(\mu) + ...) = diag\left(1 + \lambda_i R^*(\mu - \lambda_i)\right). \qquad (34)$$

Substituting (33) into (31) we get

$$\mathbf{Q}diag(1 + \lambda_i R^*(\mu - \lambda_i))\mathbf{Q^{-1}}$$
$$= \mathbf{I} + \mathbf{Q}\, diag(\lambda_i R^*(\mu - \lambda_i))\mathbf{Q^{-1}}.$$

Finally the form for the compliance moduli matrix becomes

$$\mathbf{\tilde{S}_{xy}} = \left[\mathbf{I} + \mathbf{Q}\,\mathbf{diag}(\lambda_\mathbf{i}\mathbf{R}^*(\mu - \lambda_\mathbf{i}))\mathbf{Q^{-1}}\right]\mathbf{S_{xy}}. \qquad (35)$$

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